

# ANALYSIS OF OBJECTS IN FIRE SITUATION WITH THE PFEM

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## 1 INTRODUCTION

Versatility features of the polymeric materials such as, high strength, low weight, ease processing, capability to form in complex shapes, etc. have led to their widespread industrial applications in aircraft structure, transportation vehicles, building and highway construction, maintenance and finishing products, electronic boards, bioengineering, structural materials, and many other different applications. Their behavior in fire is therefore of considerable interest because they play an important role in the ignition and growth stages of fire. In this paper, a new computational procedure for analysis of the combustion, melting and flame spread of polymers under fire conditions is presented. We follow the way of posing the fluid as well as the solid problem in a Lagrangian framework [2]. This approach allows to treat the whole domain, containing both fluid and solid subdomains which interact with each other, as a single entity and describes its behaviour by a single set of momentum, continuity and energy equations. The equations are discretized with the Particle Finite Element Method (PFEM) [3]. The PFEM, treats the mesh nodes in the fluid and solid domains as particles which can freely move and even separate from the main fluid domain representing, for instance, the effect of water drops. A finite element mesh connects the nodes defining the discretized domain where the governing equations are solved in the standar FEM fashion.

## 2 GOVERNING EQUATIONS

Let  $\Omega \subset^d$ ,  $d \in \{2, 3\}$ , be a bounded domain containing two different fluids (see Figure 1). We denote time by  $t$ , the Cartesian spatial coordinates by  $x = x_{i=1}^d$ , and the vectorial operator of spatial derivatives by  $\nabla = \{\partial_{x_i}\}_{i=1}^d$ . The evolution of the velocity  $u = u(x, t)$ , the pressure  $p = p(x, t)$ , the temperature  $T = T(x, t)$  and the species  $Y_k = Y_k(x, t)$  is governed by equations:

$$\frac{d\rho}{dt} + \rho \nabla \cdot u = 0 \quad \text{in } \Omega \times (0, T) \quad (1)$$

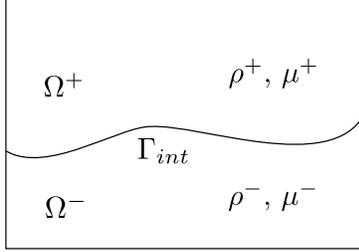


Figure 1: Two-fluid flow configuration.

$$\rho \frac{du}{dt} = \nabla \cdot \sigma + \rho f \quad \text{in } \Omega \times (0, T) \quad (2)$$

$$\rho C \frac{dT}{dt} = \nabla \cdot (\kappa \nabla T) \quad \text{in } \Omega \times (0, T) \quad (3)$$

$$\rho \frac{dY_k}{dt} = w_k + \nabla \cdot (\mathcal{D} \rho \nabla Y_k) \quad \text{in } \Omega^+ \times (0, T) \quad (4)$$

where  $\rho$  is the density,  $\sigma$  the Cauchy stress tensor,  $f$  the vector of gravity,  $C$  the heat capacity,  $\kappa$  the thermal conductivity,  $T$  the temperature,  $Y_k$  mass fraction of species  $k$ ,  $w_k$  source term of specie  $k$ ,  $\mathcal{D}$  diffusion coefficient and  $\frac{d\phi}{dt}$  represents the total or material derivative of a function  $\phi$ .

Let  $\Gamma_{int}(t)$  be the interface that cuts the domain  $\Omega$  in two open subdomains,  $\Omega^+(t)$  and  $\Omega^-(t)$ , which satisfy:  $\Omega^+ \cap \Omega^- = \emptyset$ ,  $\Omega = \bar{\Omega}^+ \cup \bar{\Omega}^-$ , and  $\Gamma_{int} = \bar{\Omega}^+ \cap \bar{\Omega}^- = \partial\Omega^+ \cap \partial\Omega^-$ . In each subdomain, the physical properties are defined as:

$$\rho = \rho(x, t) = \begin{cases} \rho^+ & \text{if } \in \Omega^+(t) \\ \rho^- & \text{if } \in \Omega^-(t) \end{cases}, \quad C = C(x, t) = \begin{cases} C^+ & \text{if } \in \Omega^+(t) \\ C^- & \text{if } \in \Omega^-(t) \end{cases},$$

$$\kappa = \kappa(x, t) = \begin{cases} \kappa^+ & \text{if } \in \Omega^+(t) \\ \kappa^- & \text{if } \in \Omega^-(t) \end{cases}, \quad \mathcal{D} = \mathcal{D}(x, t) = \begin{cases} \mathcal{D}^+ & \text{if } \in \Omega^+(t) \\ 0 & \text{if } \in \Omega^-(t) \end{cases}$$

## 2.1 CONSTITUTIVE EQUATIONS

In this paper is assumed that the polymer melt flow and the surrounding air are governed by the equations of an incompressible fluid. We treat the incompressible fluid as the limit case of a nearly-incompressible newtonian flow. The constitutive equation of a newtonian fluid is

$$\sigma = -pI + 2\mu\dot{D} \quad (5)$$

where  $\mu$  is the fluid viscosity,  $p$  is the thermodynamic (or hydrostatic) pressure of the fluid and  $\dot{D}$  is the deviatori components of the of the symmetric part of the velocity gradient tensor  $L$ . Following [1], if the newtonian flow is nearly-incompressible then density changes

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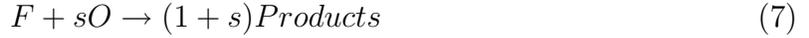
are related to pressure changes by the following relationship:

$$dp = \frac{K}{\rho} d\rho \quad (6)$$

where  $K$  is the elastic bulk modulus of the fluid.

### 3 COMBUSTION PROBLEM

In the present study, the polymer/air reactive system is modelled as a simplified one-step chemical reaction between the fuel (F) and oxidizer (O)



where  $s$  is the stoichiometric ratio[5]. These species are identified by their mass fraction  $Y$  as follows:  $Y_F$ ,  $Y_O$  and  $Y_P$ . Species reaction rates  $w_k$  are all related to the single-step reaction rate[7, 6]

$$w_m = -B \frac{1}{T^2} Y_f Y_o e^{-(E/RT)} \quad (8)$$

where  $B$ ,  $E$  and  $R$  are appropriate constants and the temperature  $T$ . So the oxidizer and product reaction rate are linked to the fuel reaction rate:

$$w_O = (s)w_m \quad (9)$$

$$w_P = (1 + s)w_m \quad (10)$$

and the heat release per unit of volume from combustion is therefore scaled according

$$Q = -w_m \Delta H \quad \text{in } \Omega^+ \quad (11)$$

where  $\Delta H$  is the heat of combustion.

### 4 TIME AND SPATIAL DISCRETIZATION

To obtain numerical solutions to equations we use a Updated Lagrangian. Applying the Backward Euler method to integrate in the time the Eqs.(1-4), multiplying these equations by the test function and integrating over the domain, the resulting nonlinear system of equations has to be solved iteratively. Therefore, a Picard iteration is used for the linearization of all equations, leading to a relatively simple fixed point type solution procedure [4].

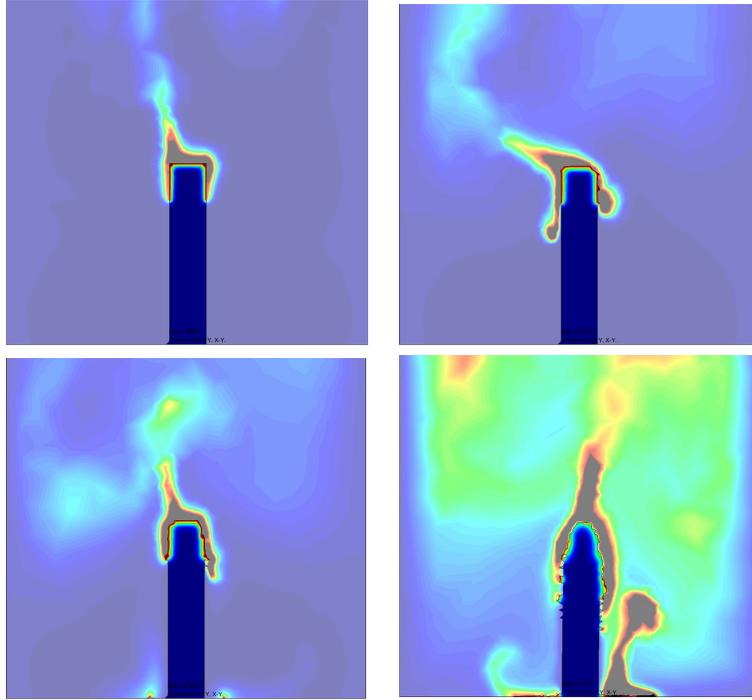


Figure 2: Melting of candle: temperature evolution

## 5 NUMERICAL EXAMPLE

In this section we show the capability of PFEM to handle interfaces undergoing topology changes due to melting, dripping and combustion. The problem consists in burning the candle inside a closed container. At the time span of  $t=0-10$  [s] the temperature at the top of the candle is to set to  $950[K]$ . In the solid phase, the processes of heating, gasification and oxidation take place. Simultaneously, in the gas phase, the chemical processes are initiated: temperature, fuel and concentration gradient of the oxidizer develop. Ignition in the gas phase occurs only if a sufficient amount of gaseous fuel is found in the reaction zone and the gas phase temperature is sufficiently high. The temperature increases in the candle due to the heat produced in the combustion process, while the viscosity decreases by several orders of magnitude as a function of temperature. This induces the melt and flow of the candle material in the heated zone. The melt flows down the heated face of the sample and drips onto a surface below. Fig.(2) shows the progressive melting of the candle exposed to the combustion heat flux and the change of the flame shape.

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