GID AS A PRE/POSTPROCESSOR OF FULLY TENSIONED/COMPRESSED STRUCTURES

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Abstract: There is a wide variety of structures that bear loads with a single stress state, all members are under compression or under tension. Moreover, equilibrium states of these structures are related with funicular/antifunicular compression/tension lines. Computational modelling of these kind of structures is complex for many reasons. Usually material behaviour or shapes are difficult to describe in many cases. Moreover, classical numerical approaches with finite element techniques become not efficient enough. And finally, the solution of these structures usually drives to a high non-linear system of equations. In this paper, we present briefly a general formulation and a developing software that solves these structures. Classical approaches like antifunicular analogy or equilibrium by pressure lines have been implemented in a computational manner. Hence this methodology is efficient and robust to deal with analysis of these structures. The program is fully integrated in the GiD Pre/Postprocessor as a graphical interface. We present results of applications in fabric, design of membranes and shells, textile membranes, pressurised and tensigrid structures.

Keywords : Cable, funicular, deployable, textile, tension, inflatable, masonry structures.

1. Introduction

All members in a net of cables are fully tensioned. Textile membranes can not bear compression loads. On the contrary a fabric arch is all under compression, cathedral domes too. These structures develop resistant mechanism based on the assembly of fully tensioned/compressed elements or materials. In the case of cables physically exist a net of structural elements. Nevertheless, in the case of fabric arches a pressure line can be draw inside the geometry to establish an equilibrium state. In both cases funicular/antifunicular lines ensures the equilibrium states of the structures. In this work we briefly present the formulation to deal with this problems and some results using GiD as pre-postprocessor.

2. Formulation

We have developed a computational method to solve equilibrium problems in fully tensioned structures using a new cable element. An elastic cable without flexure stiffness, suspended from both ends under gravity adopts a catenary's curve. For this piece of curve there is known an equilibrium equation of forces. In a generic manner we can write the equilibrium equation of a j-cable:

$$c_2^j = c_1^j + f\left(F_2^j\right) \tag{1}$$

Where c_2^j represents the nodal coordinates of the ends and F_2^j the forces on the second end.

This relation is not lineal and includes the effect of external loading by gravity. We modified this equation to take account the change of length between the original and deformed shape and hence the redistribution of total loads. Details can be found in [1]

In a net of cables we must ensure the equilibrium of an isolate j-element with equation (1). Nevertheless in all nodes of the net we also should ensure equilibrium among elements with external forces and compatibility of displacements among elements. The accomplishment of these conditions drives to set of high non-linear equations system.

$$K\left(F_2^j\left(c\right)\right) c = f \qquad (2)$$

Where K is the stiffness matrix, c nodal displacements and f external forces.

A Newton-Raphson strategy solves this non-linear system. To improve convergence and stability we found the analytical expression of a tangent stiffness matrix instead to compute it by finite differences.

3. Stability of fabric structures

Heyman [2] applied plastic theorems to the calculation of equilibrium states in fabric structures. The theory of limit analysis has become a powerful method to decide about the stability of fabric structures. Moreover it is possible to find a geometric safety coefficient for each fabric structure before to collapse.

There is a known analogy from Hooke between the shape of suspended cable and an arch both in equilibrium. Fabric develops a bearing mechanism based on a line of compression inside the arch boundary. This simple idea shifts the equilibrium state of an arch or dome to the equilibrium state of a net of cables inside the structure. We combined both principles to find equilibrium states in fabric structures.

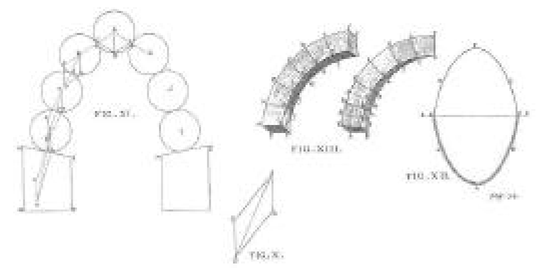


Fig. 1 Equilibrium analogy between an arch and a catenary

We verified the equilibrium of Cathedral of Mallorca applying that formulation. The funicular model is constructed with pieces of joined catenaries and loaded according the mass distribution. Results fortunately confirm the stability of the structure; however the structure is quite on the limit and safety is very low in agreement with other authors Rubio[3].

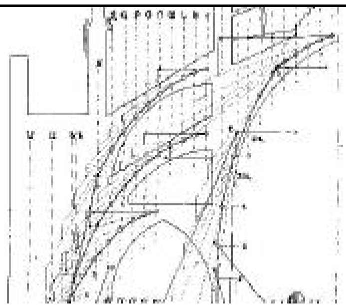


Fig. 2 Rubio's solution for the equilibrium of Cathedral of Mallorca

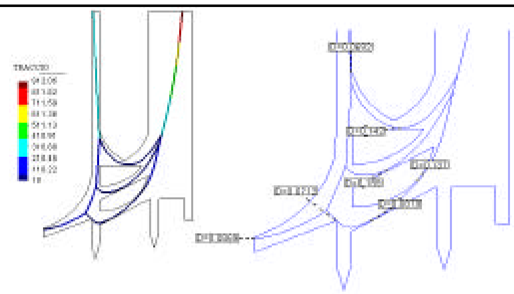


Fig. 3 Net of catenaries to model Cathedral of Mallorca

We also applied the same procedure to the Cathedral of Girona. This construction is one of the widest span in gothic style. In this case we defined a three-dimensional net of catenaries loaded with the weight of the vaults. The funicular shape is inside the vaults ensuring the equilibrium of the structure. The geometrical complexity of the model is easily overcome with graphical GiD capabilities.

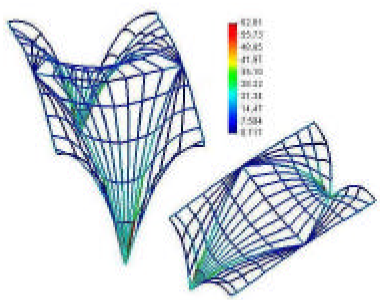


Fig. 4 Catenary's net modelling

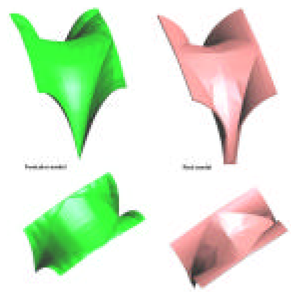


Fig. 5 Catenary's net surface is inside and similar to the shape of vaults

4. Design of shells and membranes

Famous architects like Frei, Steiner or Gaudi [4],[5] build prototypes with cables and weights to find antifunicular shapes. It is commonly accepted that this method produces beautiful shapes in space. This technique is quite old and known.

With the combination of GiD and the formulation it is possible to reproduce virtually this kind of prototypes. The method demonstrates its capability to generate shell shapes in space.

Fig. 6 Gaudi’s prototype

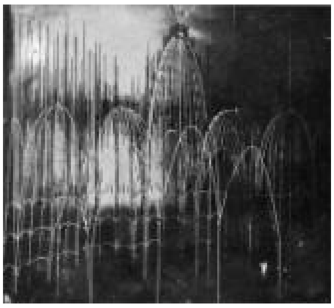


Fig. 7 Another example of Gaudi’s work

We found the shape of a roof supported only in six points. It is also possible to simulate nerves increasing the stiffness in small areas. The curvature of the shell can be controlled with the fiction elastic modulus of the cable material.

Once we get the funicular shape we can use GiD to create a geometry and compute the stresses and strains with a shell program (Calsef, ANSYS, etc.). Thus a final shape of equilibrium is found and a dimension process can start according steel or concrete normative.

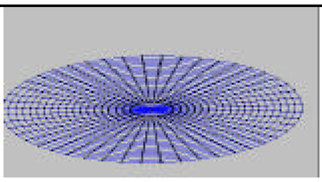


Fig. 8 **Starting geometry**

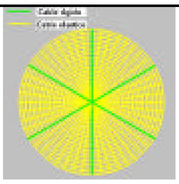


Fig. 9 **Defining nerves**

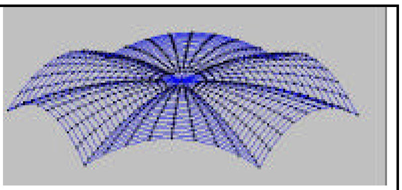


Fig. 10 **Final geometry**

5. Design minimal surfaces

Another possibility of design is to find a minimal surface as an elastic mesh that passes in a forced boundary. Under that conditions the obtained surface is minimal and smooth. The advantage is that the complexity of the boundary is always overcame by the simplicity of the method. This is also of interest for architects or designers that works with CAD tools.

A classical example is found in geometry. Scherk surface [6] is the shape of a flexible shell supported in the contour of the figures. We can find the shape of a cable net in the contour and afterwards using the graphical facilities of GiD [8] we can interpolate a continuous surface with rendering. In this case it is important to have a dense mesh enough to interpolate with guarantees.

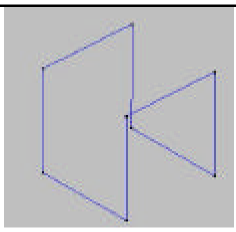


Fig. 12 Scherk contour

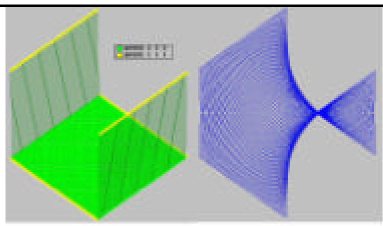


Fig. 13 Initial mesh and result

Another example is defined in figure (14-16). Two rectangular contours are joined with an elastic mesh. The final result has a high smooth regularity.

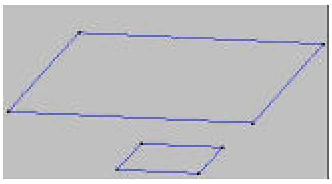


Fig. 14 Contour

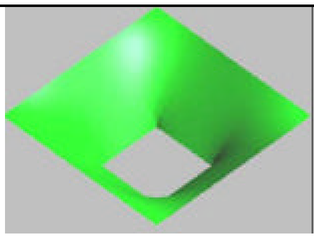


Fig. 15 Resultant surface

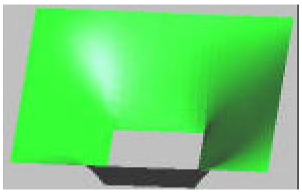


Fig. 16 Resultant surface

6. Textile membranes

Textile membranes are fully tensioned structures. The simulation in this case is more critical because this technique is capable of modelling with a good accuracy one-dimensional structural elements however textile membranes suffers two-dimensional stress states. Moreover Poisson effect is also difficult to model. However in spite of these drawbacks a first approach of the solution can be easily found. Moreover dense mesh can deal with an acceptable solution.

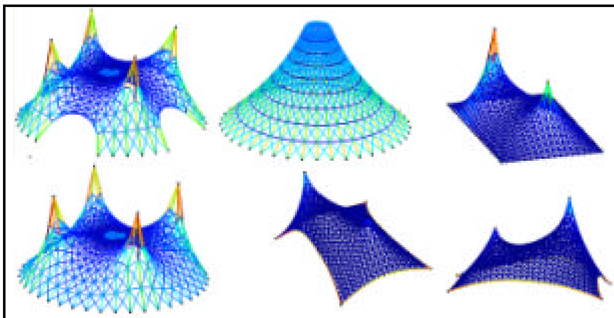
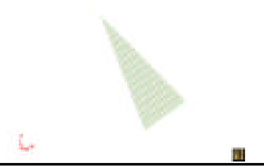
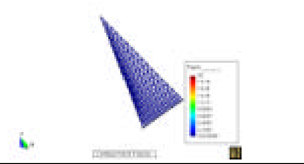
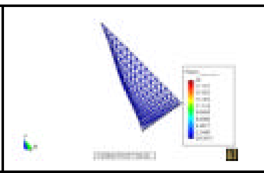
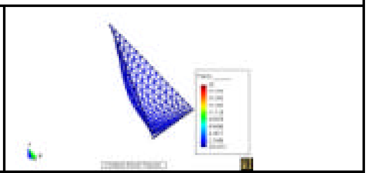
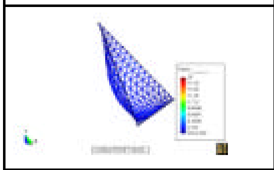
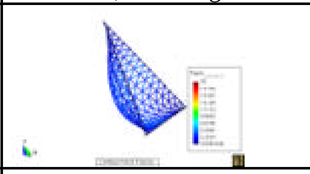
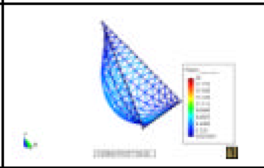
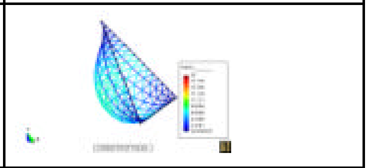
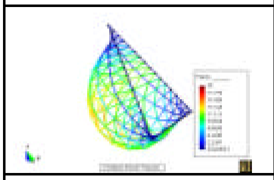
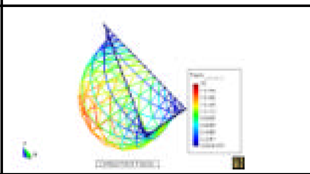
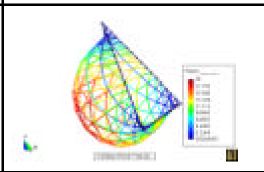
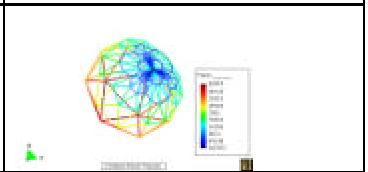


Fig. 17 Several shapes for textile membranes

7. Pressurised structures

Some structures like sails or inflatable shells suffers pressure loads. This kind of structures always have large displacements and loads must adapt to the changing surface. We tried to simulate the shape of a triangular sail under a wind. Results can be seen in figure (18).

Fig. 18. Sequence of wind effects over a sail			
			
Initial mesh	P=0, self weight	P=0.1	P=0.3
			
P=0.5	P=0.8	P=1.1	P=1.4
			
P=1.7	P=1.75	P=1.78	Diverge for critical pressure.

We also tried to simulate inflatable process of simple geometries and we found interesting shapes. In this case the load is an internal pressure always applied normally to the surface. A cones is inflated until we reach the critical pressure. Curiously shape seems an aerostatic balloon.

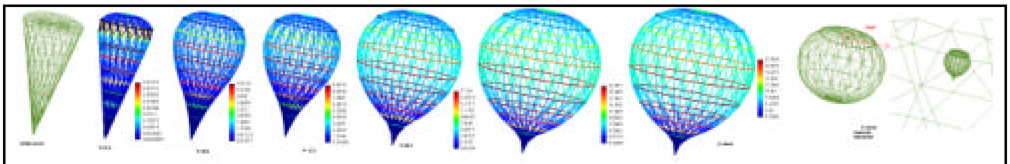


Fig. 19 An inflated balloon

In the examples we could not verify the accuracy of displacements but we could compare results with the critical pressure. Test calculated for simple shapes found perfect accuracy with numerical results.

8. Tensigrid structures

Another interesting structure is a tensigrid one: It is made of cables and bars. This structural system allows to make very slight structures in the aerospace industry for the formation of drop-down structures [7] but also roofs of large span structures. Elastic cables are the natural elements of the formulation and bars are simulated with classical approaches from matrix analysis.

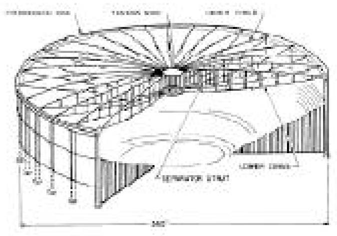


Fig. 20 Model reproducing the cover of the audience of Utica

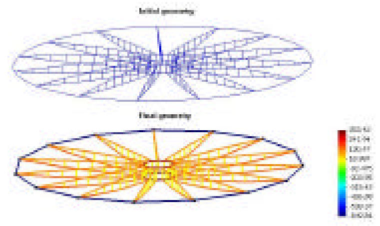


Fig. 21 Numerical simulation

9. Conclusions

In this paper we presented some results for fully tensioned/compressed structures using GiD as a pre-post processor. The methodology combines catenary's net under gravity loads with funicular/antifunicular modelling. We tested the method for fabric stability analysis and we found similar results compared with other authors. We demonstrated that this approach can help during the design process using minimal surfaces or antifunicular shells. Moreover we applied the method for textile membranes, several drawbacks of modelling can affect seriously the accuracy of results. Finally we applied the method for pressurised and tensigrid structures with promising results. GiD performed very good during the mesh generation and visualisation of results using surface interpolation and rendering.

10. References

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