SINGULARITY AND ELLIPTICAL MESH DEFORMATION TECHNIQUES

B. Cantaloube*

* ONERA Office National d'Études et de Recherches Aérospatiales
29, avenue de la Division Leclerc
92322 Châtillon Cedex - FRANCE
e-mail: cantalou@onera.fr

Key words: ALE, Mesh deformation, Singularity method, elliptical method, Adaptation

Abstract. *Two mesh deformation techniques are presented. The first one is based on an analogy with the well known singularity (or panel) methods, a 2-D and a 3-D examples are presented. The second one consists in minimizing an energy-like function in order to preserve or improve the initial mesh properties. Numerical illustrations are given on a RAE2822 airfoil.*

1 INTRODUCTION

Computation methods using the ALE (Arbitrary Lagrangian Eulerian) formulation on deformable grids are in increasingly widespread use in aerodynamics. The main applications are prediction of the aeroelastic effects of aircraft wings, helicopter rotors [1] and launch vehicles. Outside the ALE framework, aerodynamic shape optimization problems are also naturally solved using grid deformation techniques. The common feature of these problems is the grid deformation resulting from the imposed movement of certain boundaries which are generally obstacle walls. One of the intrinsic problems of existing algebraic or analogous (elastic analogy) grid deformation methods specific to an application is that they deteriorate the geometric properties of the initial grid.

We propose in this work two mesh deformation methods. The first one, described in chapter 2, is based on an analogy with the well known singularity (or panel) methods used in the eighties for potential flow calculations [2], a 3-D example of mesh deformation is given on a LANN wing. A second method, described in chapter 3, has been developed to explicitly meet some geometric requirements on the deformed mesh. It is based on solving an elliptical-like system to preserve or improve the initial geometric properties of the grid [3], [4], 2-D examples of application are given on a RAE2822 airfoil.

2 THE SINGULARITY METHOD

We consider a two-dimensional domain **D** discretized by means of a given structured mesh assumed to have good smoothness and orthogonality properties. Under imposed boundaries deformations $\mathbf{d}_\Gamma$, this domain undergoes a continuous field deformation $\mathbf{d}_f$ used to relocate the grid nodes an keeps the original grid topology which serves as a reference configuration.

On body deformable boundaries, surface source (singularities) $\mathbf{\Omega}_\Gamma$ distributions induce, by means of a regularized Biot and Savart like-equation, a continuous field of displacement $\mathbf{d}_f$.

Under the constraint that $\mathbf{d}_f$ must be equal to $\mathbf{d}_\Gamma$ on the deformable boundaries, the solution of a linear system gives the source intensity. The matrix of this linear system is symmetric and it is solved by means of an iterative conjugate gradient method.

In order to reduce the memory size and the cost of the computations, a macromesh technique is used. It consists in computing the deformation on a coarse mesh and then, the interpolation of Gordon an Hall [6] gives the deformation of all nodes. A sequential domain deformation technique is also added to compute the deformation of meshes for multi-domain industrial configurations.

Figure 1 shows a sequential four block mesh deformation resulting from the bending and from a linear spanwise torsion (2 degrees) of the LANN wing. The continuity of the field of displacements given by the regularized Biot and Savart equation provides smooth updated grids even if the local properties of the mesh (volume, skewness...) are not exactly conserved.

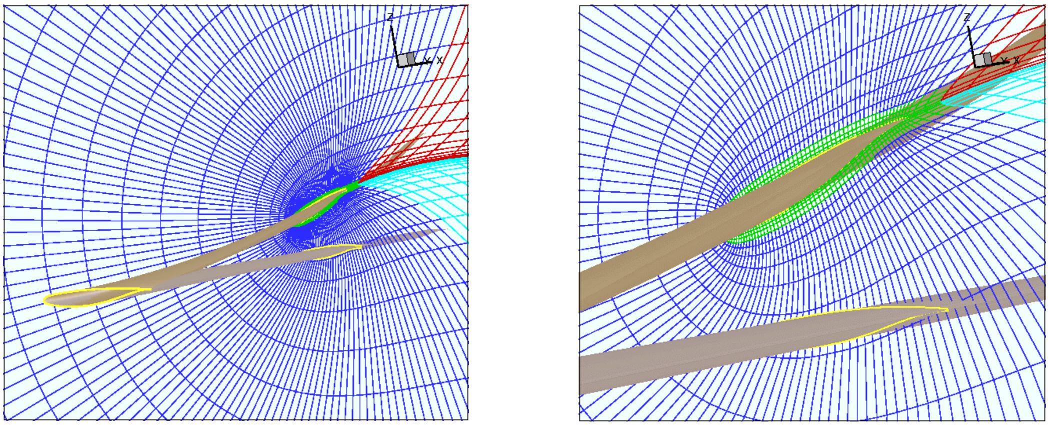

Figure 1 LANN wing. Sequential multidmain deformation

3 THE ELLIPTIC METHOD

The deformation of hyperelastic materials is governed by an internal deformation energy depending on the three invariants of the left Cauchy-Green tensor [5]. Herein, one defines an energy-like function of the vertex displacements that mimic these three invariants by conserving the length, the area (the volume in 3-D) and the orthogonality or the skewness of mesh cells, it writes :

$$E(\mathbf{d}_f) = \alpha E_l(\mathbf{d}_f) + \beta E_a(\mathbf{d}_f) + \gamma E_d(\mathbf{d}_f)\,, \tag{1}$$

where α, β and γ are arbitrary positive coefficients. The energies related to the stretching or the compression of a cell edge, to its volume and skewness variations are respectively represented by $E_l(\mathbf{d}_f)$, $E_a(\mathbf{d}_f)$ and $E_d(\mathbf{d}_f)$. The variation of the area is computed by means of the vector displacement flux through the cell interfaces. The skewness change is measured as the variation of the angle of two unit vectors.

One can easily see that, because this energy is formulated as a positive quadratic function, there exists only one global minimum. This measure of the deformation verifies the axioms of frame-indifference, isotropy and homogenous properties [3]. The minimization of this energy, with respect of all the internal node displacements $\mathbf{d}_f$, gives a linear system of equations :

$$\mathbf{K}\mathbf{d}_f = \mathbf{A}_f^T\mathbf{A}_f\mathbf{d}_f = -\mathbf{A}_f^T\mathbf{A}_\Gamma\mathbf{d}_\Gamma \tag{2}$$

All the coefficients of the matrices are known and depend only of the node coordinates. The matrix $\mathbf{K}$ is symmetric and positive. Therefore, it is numerically solved by means of an iterative conjugate gradient method. When high aspect ratio grid cells are involved, for example in the boundary layer area, a preconditionning technique is mandatory, otherwise the numerical process of a conjugate gradient or even of every other iterative methods would not converge. An efficient diagonal preconditionner $\mathbf{P}$ has been choosen in order that the matrix $\tilde{\mathbf{K}} = \mathbf{P}^T\mathbf{K}\mathbf{P}$ of the resulting linear system keeps its symmetric property and that its diagonal terms become equal to unity. This method can also address mesh optimization. The energy equation (1) can be modified in order to meet mesh properties requirements (length, volume or skewness of cells). That leads to a linear system (2) in which only the second member is modified.

For a one-dimensional configuration the second member of equation (1) reduces to its first term. This case is usefull to understand the behavior of the deformation law (2). One can demonstrate that, for high aspect ratio grids, this equation behaves as a fisrt order differential equation and for low aspect ratio it behaves like a Laplace or Poisson equation. Figure 2 illustrates this behavior on the compression of a one-dimensional beam with an high aspect ratio grid on its left end and a low one on the right. The red curve shows the deformed beam an the zoomed displacement green curve shows that the very left part moves as a solid while the right area undergoes a classical linear deformation. The deformation law (2) has the same properties in multidimensional configurations. Therefore the areas of streched grid, for example boundary layer in CFD, have solid movements similar to those of the closest deforming boundary. This property ensures to the updated grid to keep, in these regions, its initial properties.

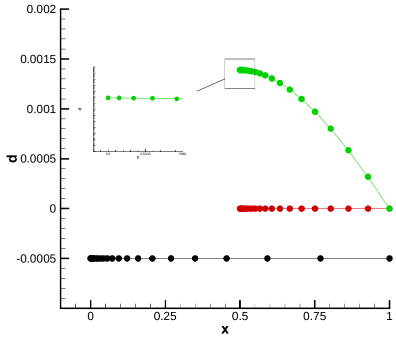

Figure 2 Asymptotic behavior of the mesh deformation law

Figure 3 shows the simulation of a 45° trailing edge flap deflection. The two enlarged areas show that even in highly deformed regions, the grid preserves its geometric original properties.

Figure 4 depicts the configuration used to assess the performances of the elliptical method for adapting a mesh in the region of an aerodynamic shock. The size aspect ratio of the first row of cells surrounding the airfoil is 10^{-5}. The shock is approximatively located at the mid chord of the profile. The deformation method is applied to obtain the updated grid (figure (4)) with prescribed cell sizes in the area of the shock. The red mesh line, crossing the shock, shows that the mesh adaptation area is independent of mesh lines, the left part of figure (4) shows that this line does not cross any mesh line.

4 CONCLUSION

Two mesh deformation methods were presented. The first one is an analogy with the well known singularity or panel methods and it is well adapted to industrial configurations. The second one is based on an elliptical method minimizing a convex energy-like function which gives directly the node displacements. In two space dimensions, this method has proven, through numerical examples, his capability to compute large amplitude mesh deformations with optimization on an high aspect ratio grid.

The next step will concern the 3-D developments of this method. The theoritical approach is the same as in 2-D configurations and is straightforward. An algorithmic work should be done to minimize memory size and CPU time requirements.

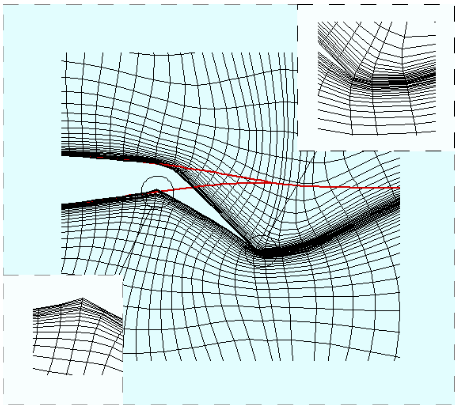

Figure 3 Simulation of the flap deflection (RAE2822 airfoil)

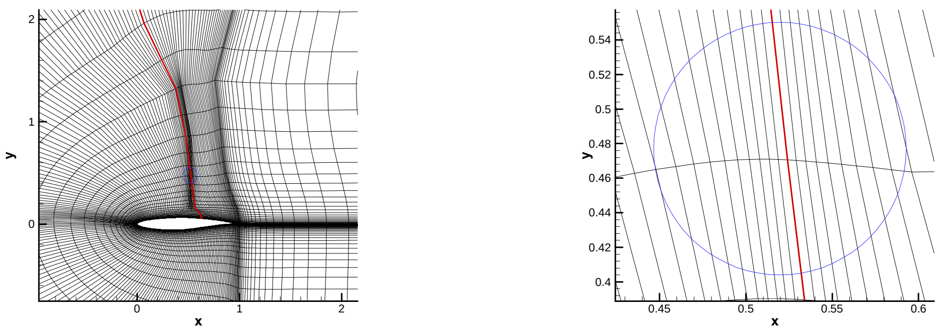

Figure 4 Mesh adaptation

REFERENCES

[1] Cantaloube, P. Beaumier, *Simulation of an unsteady aeroelastic response of a multibladed rotor in forward flight*, 27Th European Rotorcraft Forum, Moscow, Sept 2001.

[2] C. Rehbach, *Numerical calculation of 3-D unsteady flows with vortex sheets*, AIAA Paper 78-111, January 10, 1978.

[3] O. P. Jacquotte, *A new mechanical approach for grid generation in computational fluid dynamics". GAMNI/SMAI* , Automated and adaptation Mesh generation", Grenoble, octobre 1987.

[4] O. P. Jacquotte and J. Cabello, *A variational method for the optimization and adaptation of grids in computational fluid dynamics*, 2nd Int. Conference on Numerical Grid Generation in Computational Fluid Dynamics, Miami Beach, December 1988.

[5] G. Ciarlet, *Mathematical Elasticity*, Vol. I : Three Dimensional Elasticity, Stud. Math Appl. 20, Elsevier, New York, 1988.

[6] W. J. Gordon and C. A. Hall, *Construction of curvilinear coordinate systems and applications to mesh generation*, International Journal of Numerical Methods and Engineering, Vol 7, pp. 461-477, 1973.