ASSIGNMENT OF PRINCIPAL FIBERS DIRECTION TO ANALYZE ORTHOTROPIC MEMBRANES

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Abstract. *This paper deals with a methodology to analyze orthotropic out-of-plane membranes. A finite element method for a geometrically nonlinear hyperelastic analysis of membranes is used. A simple formulation in terms of the gradient deformation tensor is used for linear triangular finite elements. Transient analysis of nonlinear equations is solved using Newmark implicit time integration method combined with a full Newton-Raphson approach. Orthotropic properties of membranes are studied specially for analysis of sails. Examples problems are solved for orthotropic membranes. The assignment of a vector that defines the reinforcement direction to each finite element has no problem if the membrane is in a plane. But when a membrane structure is not in a plane as sails structures, the assignment of the reinforcement direction is not that obvious.*

1. INTRODUCTION

Membrane structures have many applications these days and are characterized because they posses only axial forces. This work is an improvement to the theory of analysis of membrane structures proposed by Taylor[i], where now it is possible to analyze orthotropic membranes Valdes[ii], Oñate[iii]. The main point in the orthotropic analysis is the right assignment of the reinforcement direction to each finite membrane element of the structure.

Suppose that a part of a sail is mesh with triangular finite elements to be analyzed using orthotropic membrane theory. Even if we work with structured meshes, the local coordinated system for each element may have different orientations as shown in Fig. 1. Note that v^1 and v^2 are the local unit base vectors in-plane of each triangular element.

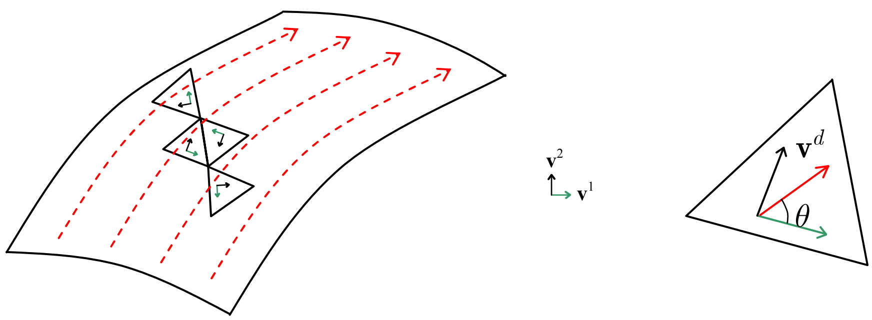

Figure 1: Principal fibers direction and element local unit base vector

2. ASSIGNMENT OF PRINCIPAL REINFORCEMENT DIRECTION

The assignment of principal reinforcement direction to a membrane is not an easy job, and we propose the following methodology. First choose any triangular element of the mesh and assign a vector $\mathbf{v}^d$ that will be its principal reinforcement direction (source element), as shown in Fig. 2. That vector will be copied to all its neighbor elements (target elements) with its corresponding in-plane direction (gray elements shown in Fig. 2).

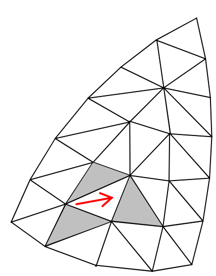

Figure 2: Element with vector of reinforcement direction

To copy $\mathbf{v}^d$ from the source element to one of the target elements we need to identify which one is the common side to both elements. From Fig. 3(a) we can see that side *i-j* is common to both triangles. Then midpoint of that side is identified as point **o**, and we build a local in-plane coordinate system at source element with the following definition. As the normal for each element is known, we place the normal for the source element $\mathbf{n}_1$ at point $\mathbf{o}$. Then axis $\mathbf{x}_1$ is built from point $\mathbf{o}$ to node j. Finally axis $\mathbf{y}_1$ is built as a Cross product of $\mathbf{n}_1$ and $\mathbf{x}_1$.

Figure 3: Copying reinforcement direction

Local in-plane coordinate system at target element is built by placing normal $\mathbf{n}_2$ at point $\mathbf{o}$. Axis $\mathbf{x}_2$ is defined as a vector going from point $\mathbf{o}$ to node i. Finally axis $\mathbf{y}_2$ is found as a Cross product between $\mathbf{n}_2$ and $\mathbf{x}_2$.

Once both local coordinate systems are defined, the next step is to choose an arbitrary point a on axis $\mathbf{x}_1$ as illustrated in Fig. 3(b). Using in-plane vector $\mathbf{v}^d$ of principal fiber direction at source element and point a, we find over axis $\mathbf{y}_1$ point b. So the vector that goes from b to a has the same direction as vector $\mathbf{v}^d$.

Finally to copy that vector to target element, the distance from point $\mathbf{o}$ to point a over axis $\mathbf{x}_1$ must be the same that from point $\mathbf{o}$ to point a' over axis $\mathbf{x}_2$, and the distance from $\mathbf{o}$ to b over $\mathbf{y}_1$ must be the same that from $\mathbf{o}$ to b' over $\mathbf{y}_2$. With point a', b' defined in-plane target element, we build a vector $a'b'$ that defines the principal reinforcement direction on target element. The same procedure is made for the other neighbor elements, and the process is repeated until all the elements have their in-plane vector that defines the principal reinforcement (fiber) direction.

3. EXAMPLE: ORTHOTROPIC SAIL

Sails may be made with orthotropic materials, and to be built, first an assembling process of single parts of the cutting pattern are sewed and glued together. Each one of the parts has fibers of principal reinforcement orientation that for optimization of the sail take the form shown in Fig. 4, where the methodology explained before has been used.

A wind pressure normal to the surface of 20 N/m^2 is applied. Membrane properties are:

Density $\rho = 2000 \text{ Kg/m}^3$

Young's modulus $E_x = 1100 \text{ N/mm}^2$, $E_y = 385 \text{ N/mm}^2$

Poisson's modulus $v_{xy} = 0.35$, $v_{yx} = 0.123$

Shear modulus $G_{xy} = 220 \text{ N/mm}^2$

Thickness $t = 0.1$ mm

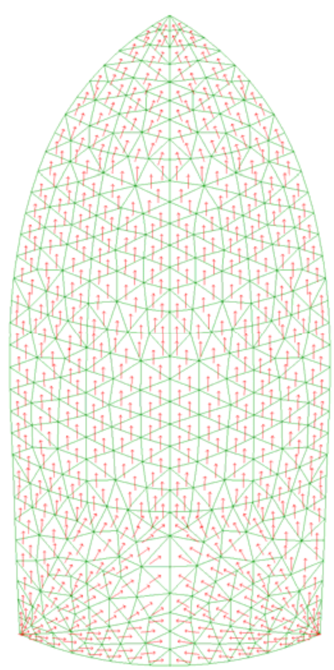
Figure 4: Orthotropic sail

Fig. 5 shows maximum displacements. An interesting solution is found when 2$^{\text{nd}}$ Piola-Kirchhoff stresses are postprocessed. In Fig. 6(a) local 2nd Piola Kircchoff stress S_{xx} is shown, where local orientation xx is drawn in Fig. 4. Local 2nd Piola Kircchoff stress S_{yy} is illustrated in Fig. 6(b) which local orientation is perpendicular to the principal reinforcement direction.

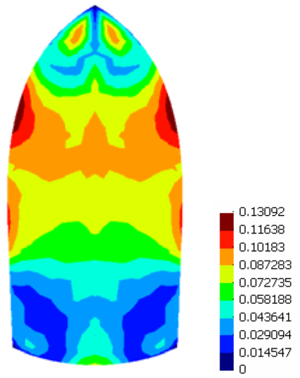

Figure 5: Sail maximum displacement [m]

Because our sail is optimized, 2nd Piola Kircchoff stresses are almost the same as principal 2nd Piola Kircchoff stresses. A more detail paper for orthotropic analysis of membranes can be found in Valdes[ii].

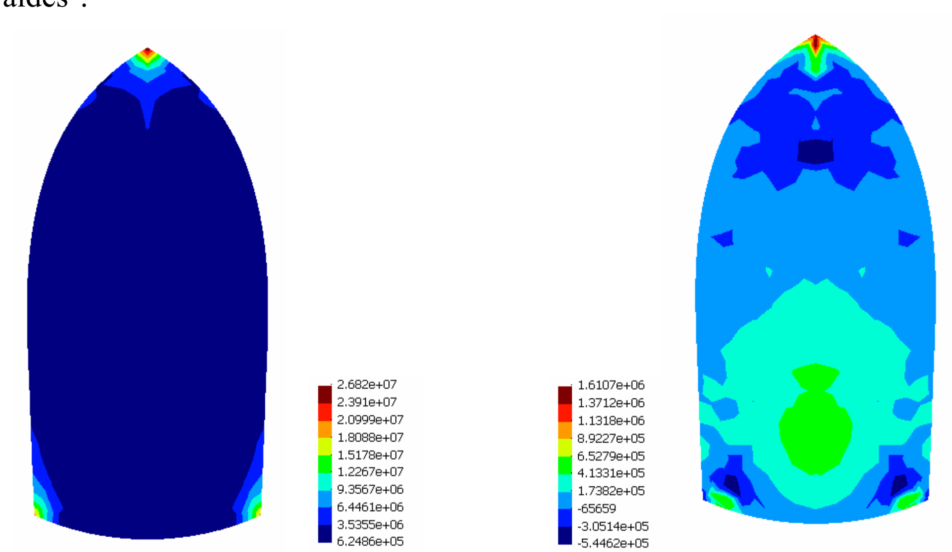

Figure 6: 2nd Piola Kircchoff stresses (a) S_{xx}, (b) S_{yy} [N/m^2]

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