FRACTALCOMS PROJECT: MESHING FRACTAL GEOMETRIES WITH GiD

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SUMMARY: Antennas based on fractal structures present some particular properties as self-similarity and miniaturization that overcome some of the main limitations of classical antennas. Defining and meshing the fractal geometry are the two first stages in the numerical analysis of their electromagnetic behavior. Fractal structures are highly convoluted with very small details, nevertheless, some of these fractals can be easily defined with very few parameters using the concept of an iterated function system (IFS). Some other important points about the meshing of fractal geometries are also described in this communication. GiD provides a very suitable environment to create arbitrary 3D meshes. One of the objectives of Fractalcoms project is to extend the capabilities of GiD in order to generate automatically the mesh of fractal geometries taking as input data the IFS parameters.

KEYWORDS: Fractals, multiband antennas, small antennas, iterated function system, adaptive meshing, numerical methods.

INTRODUCTION

The emergence of antennas with fractal geometries has given an answer to two of the main limitations of the classical antennas: the single band performance and the dependence between size and operating frequency [1]. The self-similar properties of certain fractals result in a multiband behavior of the antennas built after these fractals [2][3]. On the other hand, the highly convoluted shape of these fractals makes possible the reduction in size, and consequently in mass and volume, of certain antennas [4]. These reductions can make possible to combine multimedia, communication and teledetection funcionalities in a reduced space like a handy phone or even a wristwatch or a credit card. For instance, it has been demonstrated that a fractal antenna can provide GPS services within a conventional mobile cellular phone. The aim of Fractalcoms project is to explore the performance limits of these fractal shaped devices.

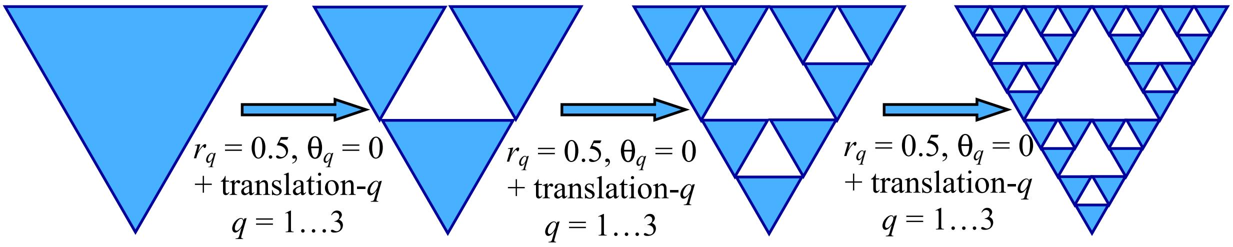

Fig. 1: Four iteration Sierpinski fractal obtainted after a set of affine transformations

Fractal structures can be analyzed using the integral equation methods (IE), in conjunction with the well-known Method of Moments (MoM) [5]. The MoM splits the fractal geometry in basis functions. Rao, Wilton and Glisson linear triangles with common vertices (RWG) [6] are usually chosen for their flexibility to model 3D arbitrary surfaces.

Since fractal structures are highly convoluted with very small details, many basis functions (unknowns) can be required in the solution of the problem. In order not to increase excessively the computational requirements a simple and smart definition and meshing of the geometry must be done.

GEOMETRY GENERATION: ITERATED FUNCTION SYSTEM

A simple way to build most fractal structures is using the concept of iterated function system (IFS) [7]. An IFS is defined by a set of Q affine transformations in the plane $\{\omega_q\}_{q=1}^{Q} : R^2 \rightarrow R^2$ which can be written as:

$$\omega_q(x) = A_q x + t_q = \begin{pmatrix} r_{q1}\cos\theta_{q1} & -r_{q2}\sin\theta_{q2} \\ r_{q1}\sin\theta_{q1} & r_{q2}\cos\theta_{q2} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} t_{q1} \\ t_{q2} \end{pmatrix} \qquad (1)$$

where x_1 and x_2 are the coordinates of point x. If $r_{q1}=r_{q2}=r_q$ with $0<r_q<1$, and $\theta_{q1}=\theta_{q2}=\theta_q$, the IFS transformation is a contractive similarity (angles are preserved) where r_q is the scale factor and θ_q is the rotation angle. The column matrix t_q is just a translation on the plane.

Applying several of these transformations in a recursive way, the self-similar fractal is obtained. In fact, self-similarity can be also understood as the property by which the fractal is found inside the fractal itself but at a smaller scale. In Fig. 1 and Fig. 2 can be found two examples of the recursive procedure followed to obtain the fractal shape. It must be noted that

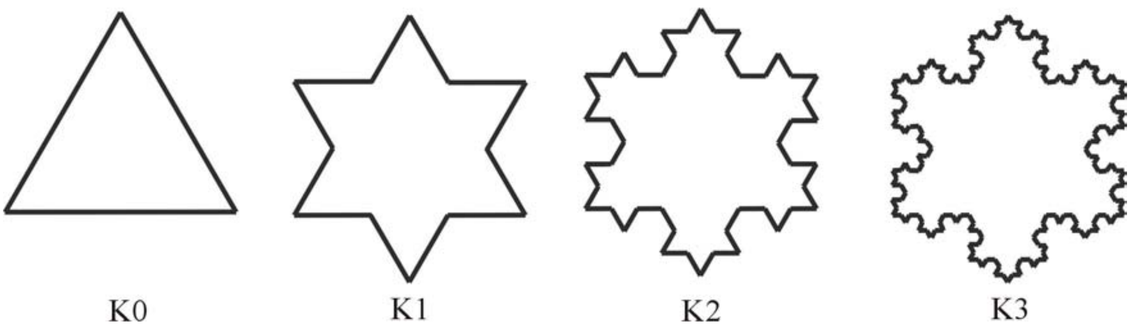

Fig 3. Koch island after three iterations

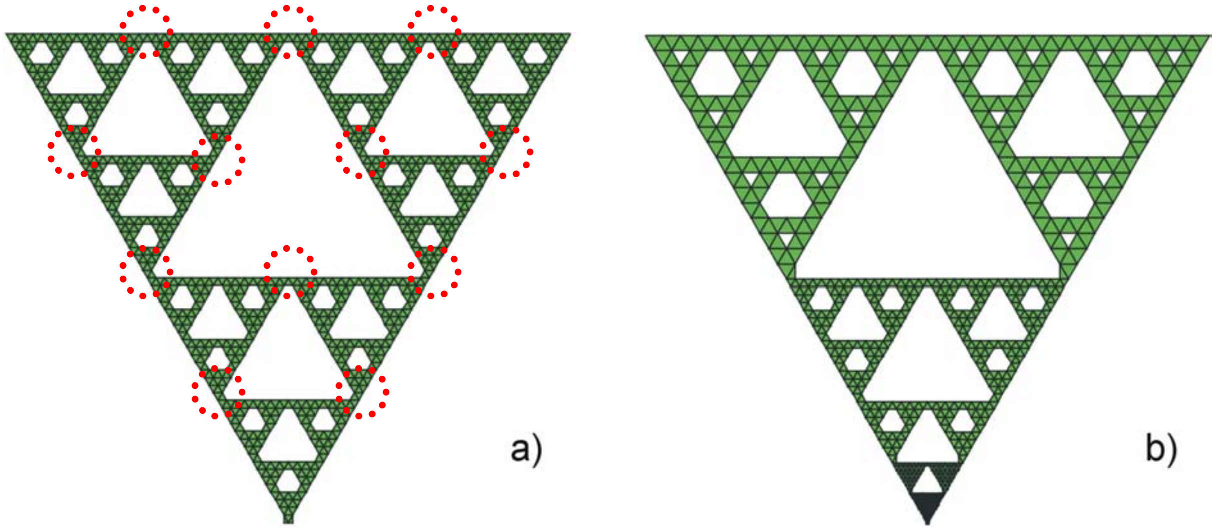

Fig. 3: a) Sierpinski gasket monopole uniformly discretized in 1700 RWG triangles.Dotted lines mark some of the small triangles joining the bigger triangles.
b) The same monopole with an adaptive meshing of 1364 RWG triangles.

the input arguments are only the initial geometry (initiator), the number of iterations and some parameters about the transformations to perform.

MESHING FRACTAL GEOMETRIES

Once the geometry has been generated, it must be descretized in RWG triangles, Fig. 3a shows a uniform discretization of a Sierpinski fractal. In Fig. 3a, it must be also noted some small additional triangles joining the larger ones. These small triangles may be required for certain initiators to be able to represent the electric density of current flowing along the whole antenna.

It is important to point out that, when applying MoM, memory storage is proportional to N^2, while computation time is proportional to N^3 where N is the number of unknowns. Then, a large number of basis functions (unknowns) will result in a large linear system that can easily overcome the capabilities of our computer.

A way to reduce these computational requirements is to take advantage of the fact that the electric density of current is concentrated in certain regions to perform an adaptive meshing. Fig. 3b shows as a finer mesh can be used in those regions with high density of current while the rest of the structure is represented with a coarser mesh. The loss of accuracy in the final results is not significant but the saving of time and memory can be very important.

CHALLENGES

GiD has been developed by CIMNE, one of the partners of Fractalcoms project, and provides a very suitable environment to create general 3D meshes. One of the objectives of Fractalcoms project is to add new features to GiD in order to create the fractal geometry and the adaptive mesh taking as input data the IFS parameters and the initiator. An appropiate mesh will decrease the number of unknowns of the problem and thus the computational requierements will be also reduced.

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